Class October 25

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$$-(py')' + qy + r = 0$$

$$-(p(z^\alpha)')' + qz^\alpha + r = 0$$

$$y = z^\alpha \qquad z = y^{1/\alpha}$$

$$Ly_1 = r$$

$$Ly_2 = r$$

$$L(y_1 - y_2) = 0$$

$$y = \sum \alpha_i y_i + y_p$$

$$Ly_p = r$$

$$\int_a^b k(x,y)\, u_\alpha(y)\, dy = \lambda u_\alpha(y) + f(x)$$

$$\int^b k(x,y)\left[u_1(y) - u_2(y)\right] dy = \lambda(u_1 - u_2)$$

$$u^i(x) = u^i_p(x) + \sum c_\alpha u_\alpha(y)$$

$$\sum_i k(x_j, y_i)\, u(y_i) = \lambda u(x_j)$$

$$K_{ji}\, u_i = \lambda u_j$$

$$\boxed{K u = \lambda u}$$

$$\sum_{l=0}^{N} K_{il} u_l = \lambda u_i$$

$$\Rightarrow \sum_{l=0}^{N} \left(K_{il} - \lambda \delta_{il}\right) u_l = 0$$

$$\det(K - \lambda I) = 0$$

$$Ku = F$$

if $\det K \neq 0$, then

$$u = K^{-1}F = K^{-1}F$$

$$Ku = \lambda u + F$$

$$(K - \lambda I)u = F$$

λ_α

$$Ku = \lambda_\alpha u \qquad \alpha = 0, \ldots, N$$

$\lambda \neq \lambda_\alpha$

$$u = (K - \lambda I)^{-1} F$$

$$Lu + cu = 0$$

$$L = a\frac{d}{dz^2} + b\frac{\wedge}{dz} + c$$

$$z = x + iy$$

Choose

$$K(z,w)$$

$$c\,u(z) = c\int_C K(z,w)\,v(w)\,dw$$

known and chosen

$$Lu(z) = L\int_C K(z,w)\,v(w)\,dw$$

$$= \int_C \big(L\,K(z,w)\big)\,v(w)\,dw$$

$$Lu + cu = 0$$

$$\int \left[L\,K(z,w) + c\,K(z,w)\right] v(w)\,dw = 0$$

$$\int_C$$

$$M_w K(z,w) = L_z K(z,w)$$

big trick

$$0 = \int_C [M_w K(z,w) + c\, K(z,w)]\, v(w)\, dw$$

26 Oct.

$$u'' = x^2 u' + xu + \sin x$$

$$p(x) = x^2 \qquad q(x) = x$$

$$r(x) = \sin x$$

$$k(x,y) = p(y) + (x-y)[q(y) - p'(y)]$$

$$= y^2 + (x-y)[y - 2y] = y^2 + (x-y)(-y)$$

$$= 2y^2 - xy$$

$$f(x) = u(0) + x u'(0) + \int_0^x (x-y) \sin y \, dy$$

$$= u(0) + x u'(0) + x - \sin x$$

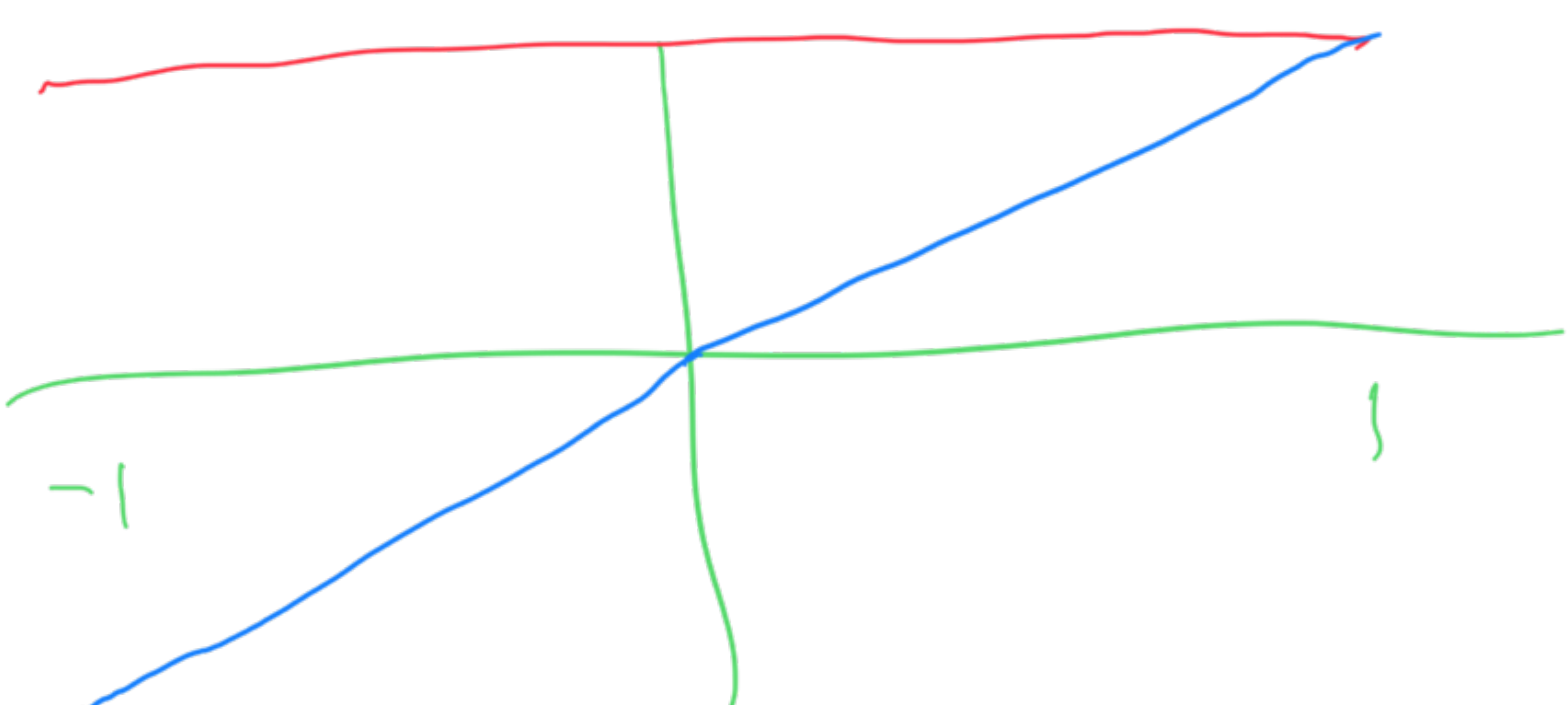
$$u(x) = f(x) + \int_0^x (2y^2 - xy)\, u(y)\, dy$$

$$u(x) = u(0) + x u'(0) + x - \sin(x) + \int_0^x (2y^2 - xy)\, u(y)\, dy$$

$$P_n(1) = 1$$

$$\int_{-1}^{1} dx\, P_n(x) P_m(x) = 0$$

$n \neq m$



$$P_2(x) = a_0 + \cancel{a_1 x} + a_2 x$$

$$1 = a_0 + a_2$$

$$0 = \int_{-1}^{1} P_2(x)\, dx = 2a_0 + a_2 \frac{2}{3}$$

$$a_2 = -3a_0$$

$$\delta(x-y) = \rho(x) \sum_i u_i(x)\, u_i(y)$$

$$\delta_{ij} = \int_a^b \rho(x)\, u_i(x)\, u_j(x)\, dx$$

$$\frac{2}{2m+1} \delta_{nm} = \int_{-1}^{1} P_n(x) P_m(x)\, dx$$

$$-\left[(1-x^2) P_n'\right]' = n(n+1) P_n(x)$$

$$\rho(x) = 1$$

$$\delta_{nm} = \int_{-1}^{1} \frac{P_n(x)}{\sqrt{\frac{2}{2n+1}}} \frac{P_m(x)\, dx}{\sqrt{\frac{2}{2m+1}}}$$

$$\delta(x-y) = 1 \sum_{n=0}^{\infty} \frac{2n+1}{2} P_n(x) P_n(y)$$

$$P_2(x) = \frac{1}{3}\left[3x^2 - 1\right]$$

$$g(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} t^n P_n(x)$$

$$G(\vec{r}-\vec{r}') = \frac{1}{|\vec{r}-\vec{r}'|}$$

$$= \frac{1}{\sqrt{\vec{r}^{\,2} - 2\vec{r}\cdot\vec{r}' + \vec{r}'^{\,2}}}$$

$$= \frac{1}{\sqrt{r^2 - 2rr'\cos\theta + r'^2}}$$

$$G(R-r) = \frac{1}{\sqrt{R^2 - 2Rr\cos\theta + r^2}}$$

$$= \frac{1}{R} \frac{1}{\sqrt{1 - 2\frac{r}{R} + \left(\frac{r}{R}\right)^2}}$$

$$= \frac{1}{R} \; g(t, x)$$

$$t = \frac{r}{R} \qquad x = \cos\theta$$

$$\phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(y)\, d^3y}{|x - y|}$$

$$g(t, 1) = \frac{1}{1-t} = \sum t^n = \sum P_n(1)\, t^n$$

$$P_n(1) = 1$$

$$g(t, 0) = \frac{1}{\sqrt{1+t^2}} = \sum P_{2n}(0)\, t^{2n}$$

$$P_{2n+1}(0) = 0$$

$$g(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum t \, P_n(x)$$

$$(n+1)P_{n+1} \quad -2nxP_n \quad +(n-1)P_{n-1}$$

$$= xP_n \quad - P_{n-1}$$

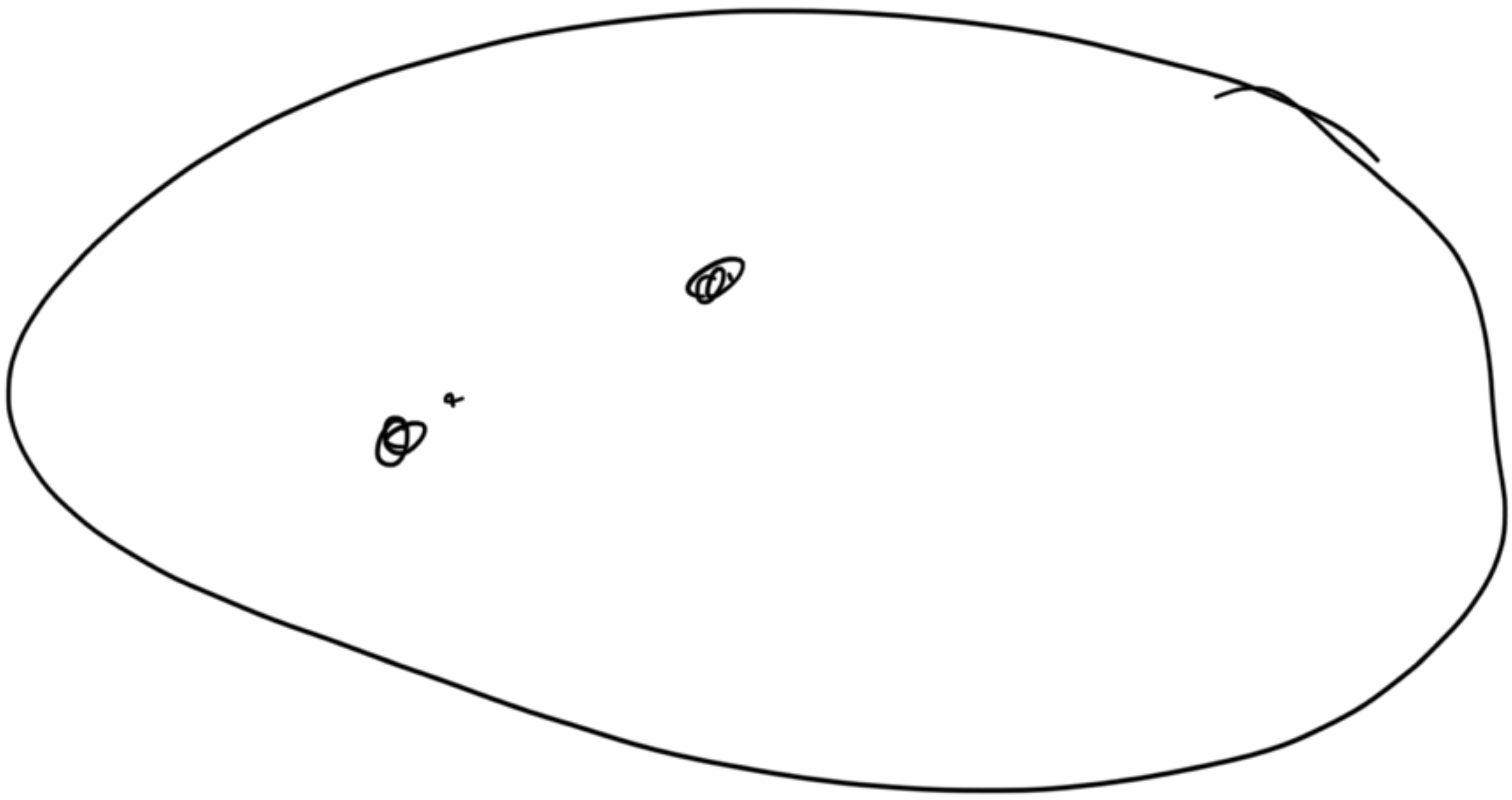
$$P_{n+1} = \frac{1}{n+1}\Big[2xnP_n + xP_n - P_{n-1} - (n-1)P_{n-1}\Big]$$

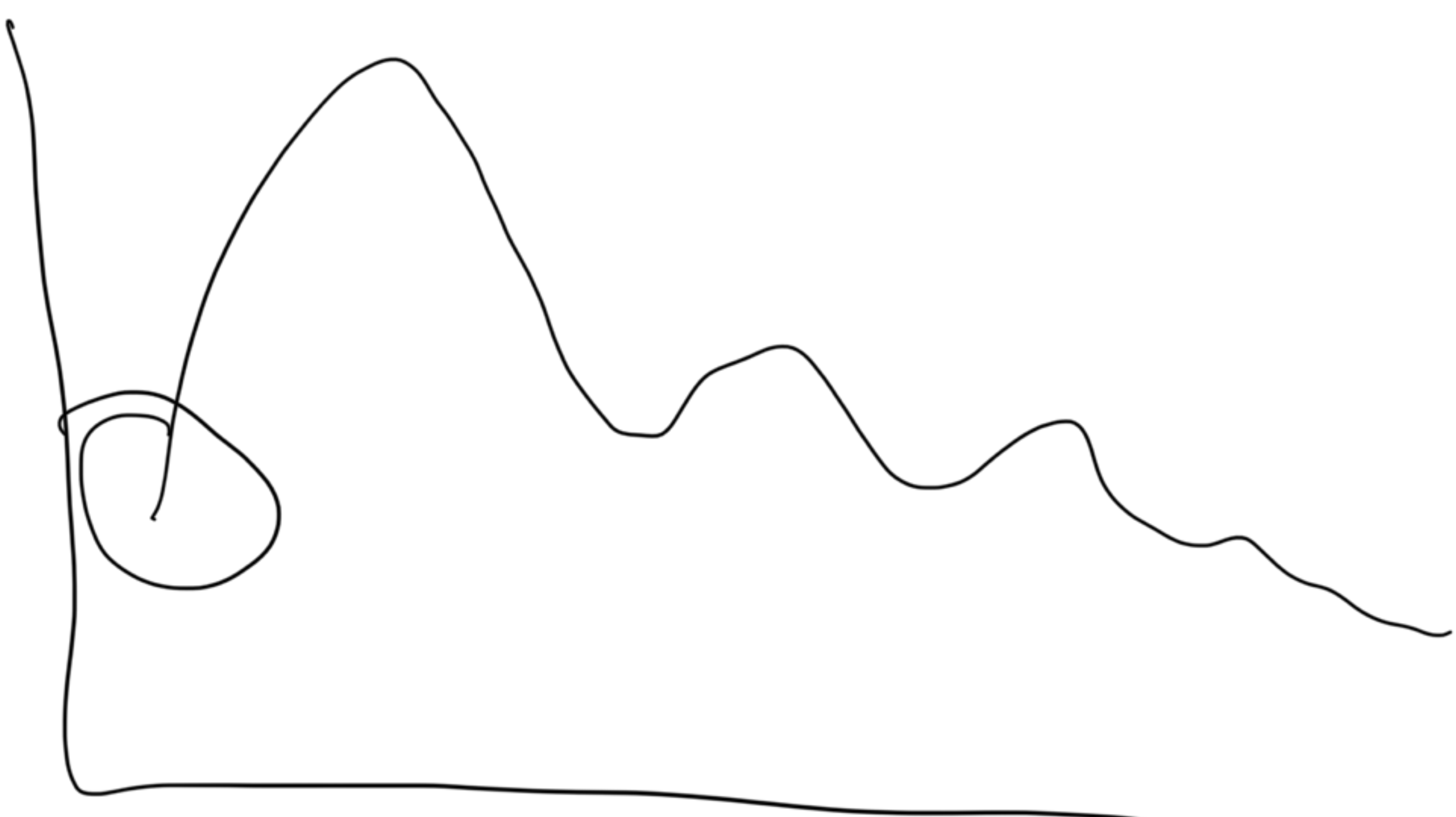
$$= \frac{1}{n+1}\Big[(2n+1)xP_n - nP_{n-1}\Big]$$

$$g(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = g(-t,-x)$$

$$c(\theta,\phi) = \sum a_{lm} Y_{lm}(\theta,\phi)$$

$$\langle Y(\theta,\phi)\, Y(\theta',\phi') \rangle$$





$$r = \frac{L}{d}$$

$$x^2 j'' + 2x j' + x^2 j = \ell(\ell+1) j$$

$$j_\ell'' = -\frac{2}{x} j_\ell' - j_\ell + \frac{\ell(\ell+1)}{x^2} j_\ell$$

$$p(x) = -\frac{2}{x}$$

$$q(x) = \frac{\ell(\ell+1)}{x^2} - 1 \qquad r = 0$$

$$h(x,y) =$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho_0}{3} \frac{1}{a^{3(1+w)}} \quad \leftarrow \frac{\dot a^2}{a^2}$$

$$\dot a^2 = \frac{8\pi G\rho_0}{3} \frac{1}{a^{1+3w}}$$

$$\dot a = \sqrt{\frac{8\pi G\rho_0}{3} \frac{1}{a^{1+3w}}}$$

$$= \sqrt{\frac{8\pi G\rho_0}{3}} \frac{1}{a^{1/2 + 3w/2}}$$

$$a^{\frac{1}{2}+\frac{3w}{2}} da = \sqrt{\quad} \, dt$$

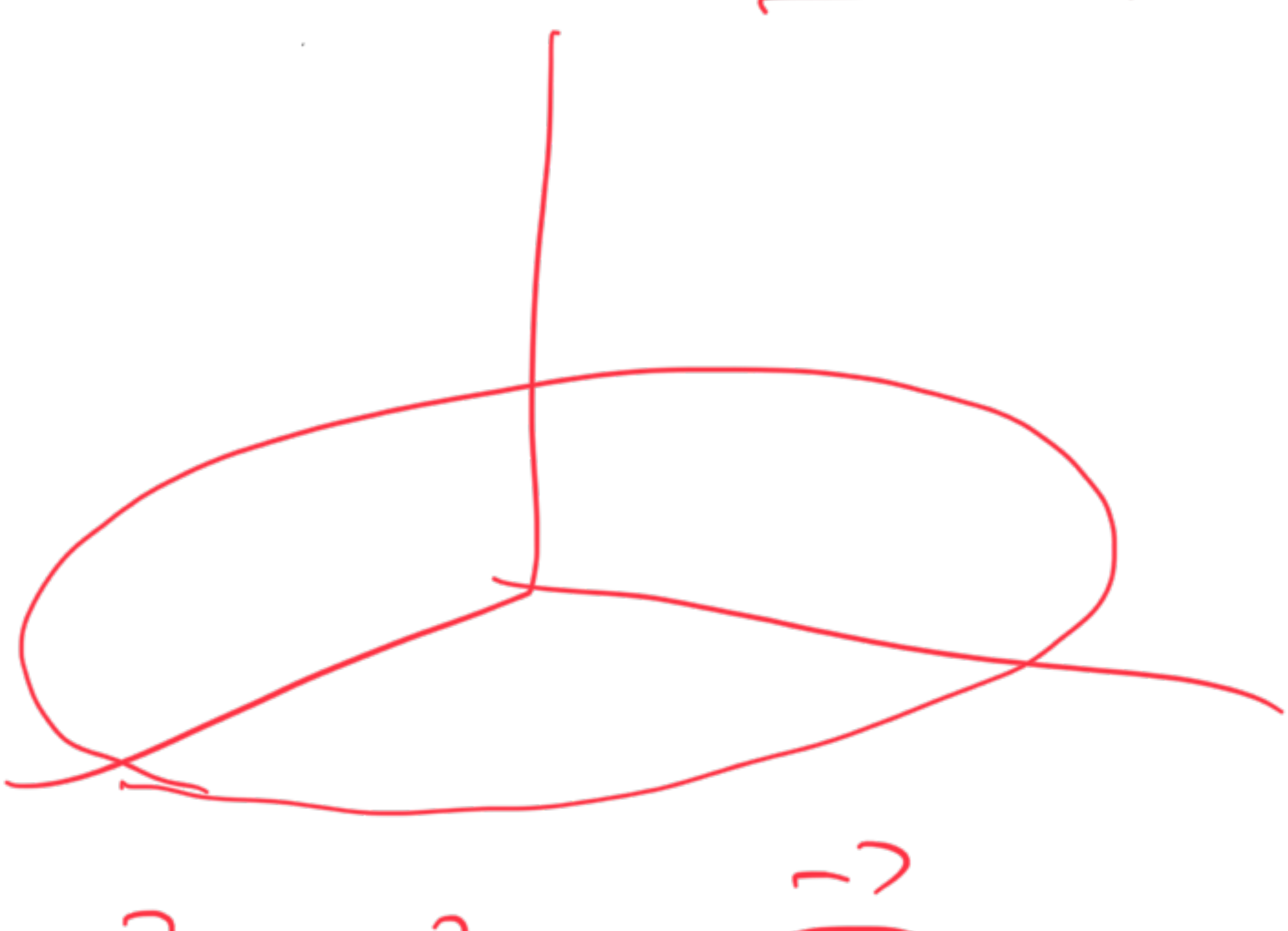
$a(0) = 0$

$\vec{B}$ of current loop in x-y plane

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A}(\vec{x}) = \mu_0 \int \frac{d^3y \, \vec{J}(\vec{y})}{4\pi |\vec{x} - \vec{y}|}$$

$$\vec{A}(r,\theta) = \hat{\varphi} \underline{\underline{A_\varphi(r,\theta)}}$$



$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\mu_0 \vec{H} = \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla \times \frac{(\nabla \times A)}{\mu_0} = \vec{J}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$$

$$A_\phi = R(r)\,\Theta(\theta)$$

$$\nabla \times (\nabla \times \hat{\phi} A_\phi)$$

$$= \hat{\phi}\left[-\frac{\partial^2 A_\phi}{\partial r^2} - \frac{2}{r}\frac{\partial A_\phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2 A_\phi}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial}{\partial \theta}(\cot\theta\, A_\phi)\right]$$

$$A_\phi = R(r)\,\Theta(\theta)$$

$$r^2 R'' + 2rR' = n(n+1)R$$

$$\Theta'' + \cot\theta\,\Theta' + n(n+1)\Theta = \frac{\Theta}{\sin^2\theta}$$

$$\Theta(\theta) = P_{n,1}(\cos\theta)$$ from Eq. (9.106)

$$r^2 R'' + 2rR' = n(n+1) R$$

$$R(r) = r^\alpha$$

$$\alpha(\alpha-1) r^\alpha + 2\alpha r^\alpha = n(n+1) r^\alpha$$

$$\alpha(\alpha+1) = n(n+1)$$

$$\alpha = \begin{cases} n \\ -n-1 \end{cases}$$

So inside the loop, i.e., for

$$r < a$$

$$A_\varphi(r,\theta) = \sum_{n=1}^{\infty} b_n \left(\frac{r}{a}\right)^n P_{n,1}(\cos\theta)$$

$$r > a$$

$$A_\varphi(r,\theta) = \sum c_n \left(\frac{a}{r}\right)^{n+1} P_{n,1}(\cos\theta)$$

Parity, Eq. (9.116)

$$P_{\ell,m}(-x) = (-1)^{\ell+m} P_{\ell,m}(x)$$

says that here $\ell + m = \ell + 1$ must be even.

So here ℓ must be odd, $\ell = 2m+1$

$$A_\ell = \sum c \left(\frac{a}{r}\right)^{2m+1} P_{2m+1,1}(\cos\theta)$$

$$n_\ell(x) = (-1)^\ell x^\ell \left(\frac{1}{x}\frac{d}{dx}\right)^\ell \left(-\frac{\cos x}{x}\right)$$

Second ~~kind~~

$$\frac{(\sin\theta\,\Theta')'}{\sin\theta\,\Theta} - \frac{m^2}{\sin^2\theta} =$$

$$\sin\theta(\sin\theta\,\Theta')' - m^2\Theta =$$
$$-\ell(\ell+1)\sin\theta\,\Theta$$

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\, J_{\ell+\frac{1}{2}}(x)$$

$$J_n \sim z^n$$

$$j_\ell \sim \frac{1}{\sqrt{x}}\, x^{\ell+\frac{1}{2}} \sim x^\ell$$

$$J_n(z) \quad z^n \sum z^{2m}$$

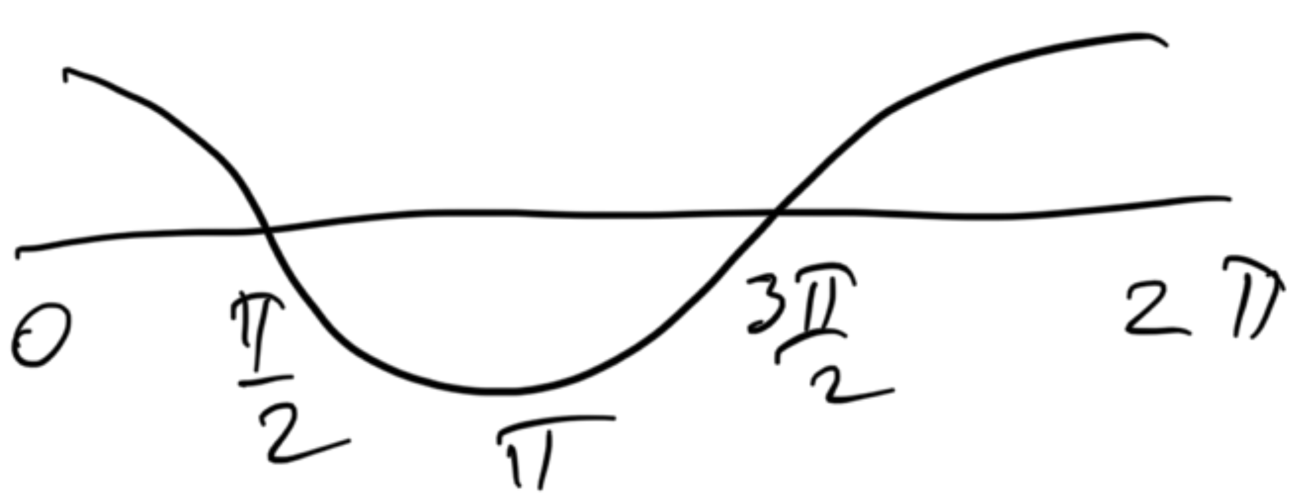
$$J_n(-z) = (-1)^n J_n(z)$$

$$e^{iz\sin\theta} + e^{-iz\sin\theta}$$

$$J_n(z_{n,m}) = 0$$

$$m = 1, 2, \ldots$$

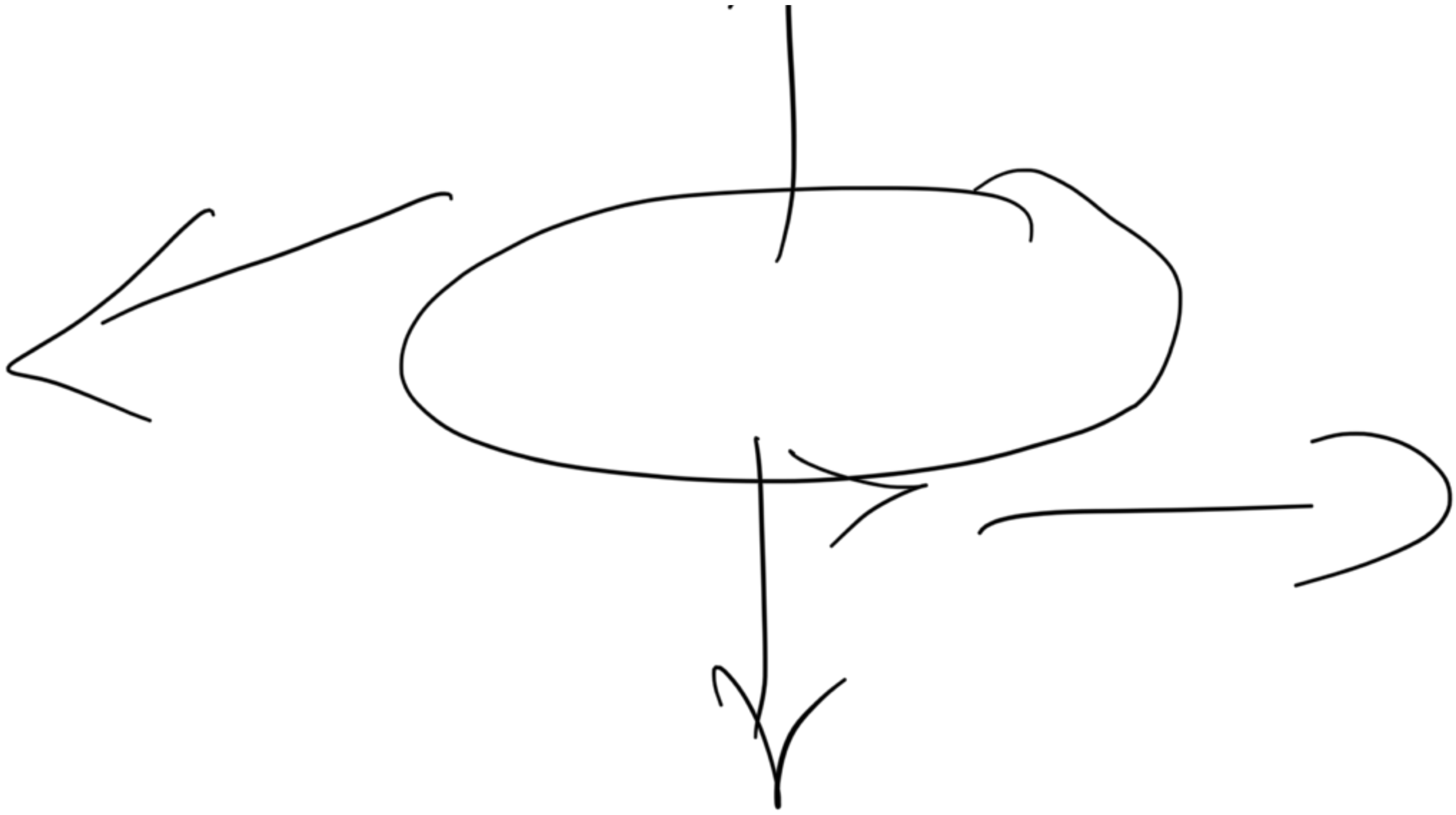
$$\cos(x) = 0 \qquad x = \frac{2m+1}{2}\pi$$



$$z - n\frac{\pi}{2} - \frac{\pi}{4} = \frac{2m+1}{2}\pi$$

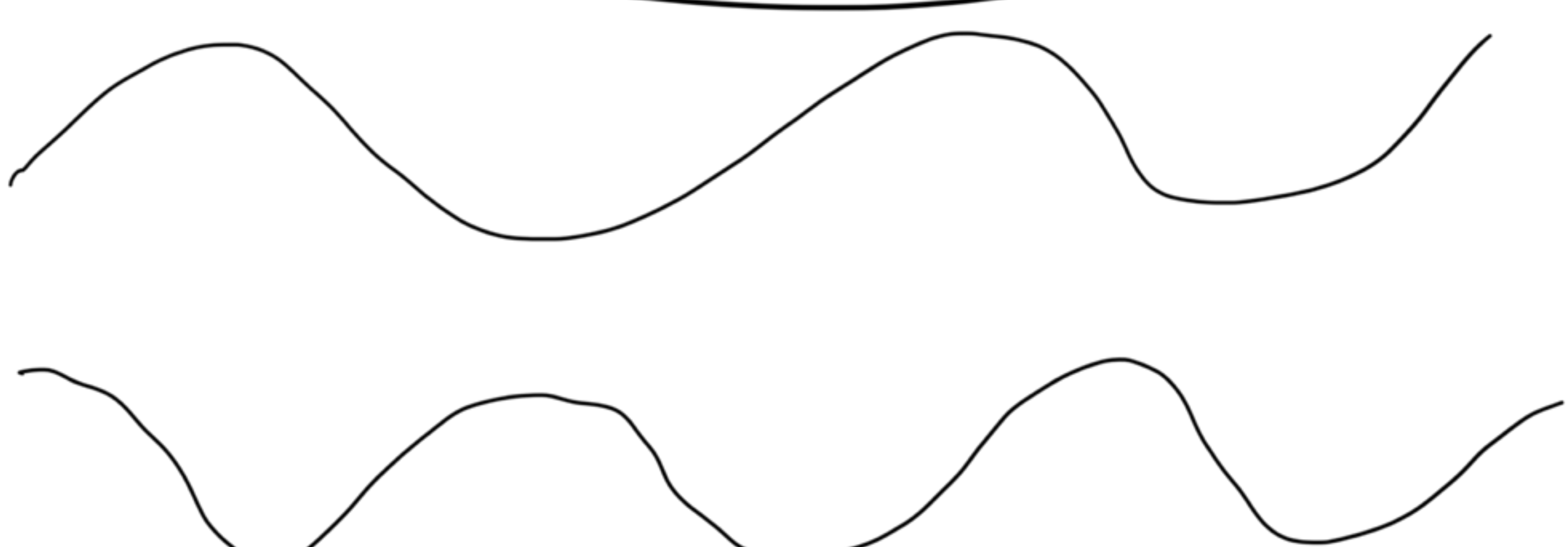
$$z = \frac{2m+1}{2}\pi + n\frac{\pi}{2} + \frac{\pi}{4}$$

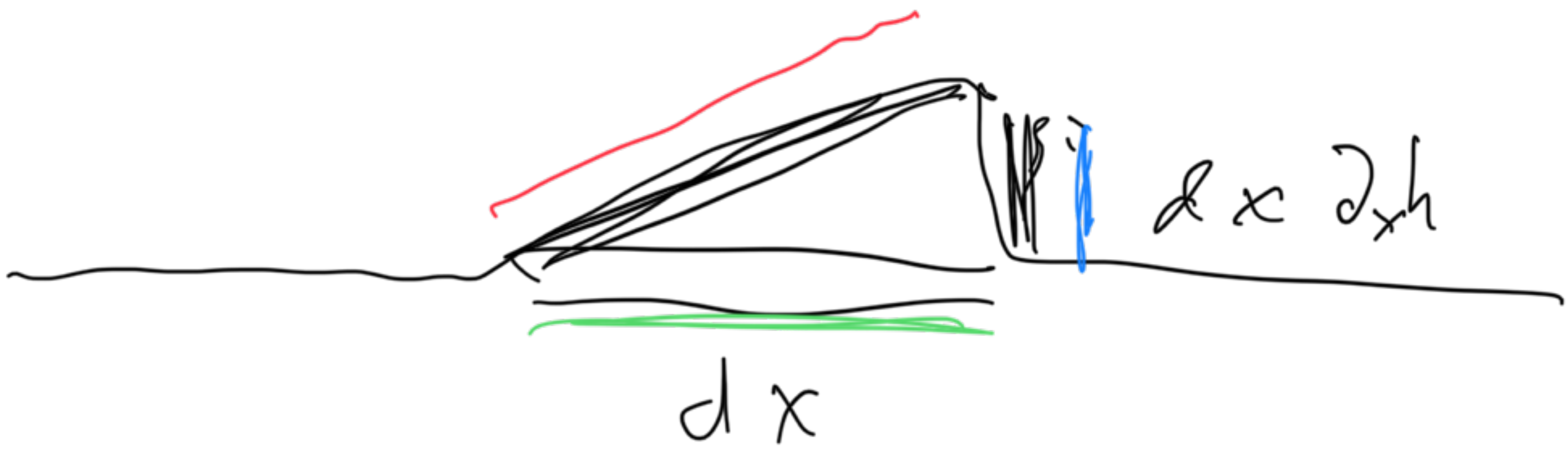
ANR.



$$\frac{e^{in\theta} - e^{-in\theta}}{2i} = \sin\theta$$

$$\frac{e^{in\theta} + e^{-in\theta}}{2} = \cos\theta$$





$$\sqrt{dx^2 + dh^2}$$

$$= \sqrt{dx^2 + (dx)^2\left(\frac{\partial h}{\partial x}\right)^2}$$

$$= \sqrt{1 + h_x^2}\, dx$$

$$\mu h_{tt} = \sigma\left(h_{xx} + h_{yy}\right)$$